

A Differential Volume-Redshift Test

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ABSTRACT

The geometry of Freedman-Roberston-Walker cosmological models is fixed by the mass density parameter, Ω_M , and the cosmological constant, Ω_Λ . The classical volume-redshift cosmological relation is a sensitive $\Omega = [\Omega_M, \Omega_\Lambda]$ indicator but its redshift dependence is observationally degenerate with the luminosity or number density evolution of galaxies. Introducing a measurement of the invariant co-moving mass density of the universe reduces the problem of galaxy evolution to a differential measurement between clustered and field galaxies. The cost is a 25% reduction in sensitivity to the Ω 's, although this test still remains 50% more Ω sensitive than the magnitude-redshift relation. An implementation of the test as the product of the mass-to-light ratio, M/L , of some clustered systems such as galaxy groups or clusters, with j/ρ_c , the normalized luminosity density, is considered. Over the zero to one redshift range the apparent $\Omega_e(z) = M/L \times j/\rho_c$ has a zero point and slope related to Ω_M and Ω_Λ , respectively. All quantities are used in a differential sense, so that common selection effects, dynamical scale errors, and galaxy evolution effects will largely cancel. The residual differential galaxy evolution between field and the clustered galaxies can be measured from the sample data. Monte Carlo simulations, calibrated with observational data, show that 20 clusters spread over the zero to one redshift range, each having 100 cluster velocities, allows a 99% confidence discrimination between open and closed low density universe models. A similarly distributed sample of about 100 rich clusters, or about 1000 galaxy groups found within a large field survey, will measure Ω_Λ to about 7% statistical error.

1. Introduction

The existence of a nonzero cosmological constant, Λ , would have profound significance for our understanding of the universe and its physics (Weinberg 1997). The expansion history and geometry of the universe, as described by the Freedman-Roberston-Walker (FRW) solution, are completely determined given the density parameter, $\Omega_M = \rho_0 8\pi G/3H_0^2$, and the cosmological parameter, $\Omega_\Lambda = \Lambda c^2/3H_0^2$. Knowledge of $\Omega = [\Omega_M, \Omega_\Lambda]$ is also of practical concern to interpret the physical properties of objects at large redshifts. The geometrical effects of the cosmological parameters are the basis of a number of classical tests of the world model. These include the redshift dependence of galaxy numbers, sizes or luminosities (*e.g.* Sandage 1961a, Peebles 1993). The success of any of these tests is in large part dependent on the degree to which the evolution of the intrinsic properties of galaxies is understood so that those effects can be removed to leave the cosmological variation of interest (*e.g.* Sandage 1961b, Tinsley 1968, Sandage 1988, Ostriker & Hausman 1977, Tremaine & Richstone 1977). The number count magnitude relation has long been taken as a hint that the $\Omega_M = 1$ model might not be correct, but this remains locked in the controversies of galaxy formation (*e.g.* Loh & Spillar 1986, Koo & Kron 1992, Ellis *et al.* 1996, Cowie *et al.* 1996, Fukugita *et al.* 1990, Shimasaku & Fukugita 1997). Although both the observational and theoretical understanding of galaxy evolution is advancing rapidly the fundamental degeneracy between galaxy evolution measurements and the cosmological parameters means that to derive a reliable empirical model requires additional information.

There are a number of alternate approaches designed to establish observational constraints on the value of Ω_Λ . One geometrical test is to compare the redshift and angular extensions of some physically understood shape at a large redshift, such as the quasar-quasar correlation function (Alcock & Paczynski 1979, Matsubara & Suto 1996, Ballinger, Peacock & Heavens 1997, Popowski *et al.* 1997). The optical depth for multiple gravitational images of distant quasars increases rapidly with positive values of Ω_Λ . The relatively low frequency of split images argues that $\Omega_\Lambda \lesssim 0.7$ (Carroll, Press & Turner 1992, Maoz & Rix 1993, Kochanek 1996). The high precision of photometry coupled with the growing understanding of supernovae, particularly those of type Ia (Hamuy *et al.* 1993, Riess, Press & Kirshner 1996), allows the classical magnitude-redshift test to be implemented. With sufficient data over a large redshift range (Perlmutter 1997, Schmidt *et al.* 1996) both the Ω values can be measured. Of particular note is the “first Doppler peak” in the angular fluctuation spectrum of the Cosmic Background Radiation which is a measurement over the largest possible path length of the geometry of the universe (*e.g.* Bond 1996 and references therein).

The cosmological parameters are of sufficient importance that they will be measured with a variety of independent methods to establish their values with confidence, understand the astrophysics of the objects, and, to some degree test the FRW model itself. The purpose of this paper is to note a variant of the classical volume-redshift test which breaks the cosmology-galaxy evolution degeneracy of the volume-redshift relation. That is, the co-moving mass density, which is an invariant at low redshift in all conventional cosmologies, is equal to the product of the total

mass-to-light ratio, M/L , with the field luminosity density, j (Oort 1958). The virialized systems can range from rich clusters to small groups of galaxies. Virialized systems have the considerable benefit that their mass profiles can be inferred from dynamical techniques, independent of the distribution of the galaxies. Furthermore dynamical mass measurements are distance independent (other than the cosmological factors of interest) with no corrections required to compare masses at different redshifts.

The following section briefly reviews the volume-redshift relation in the FRW models. The relations are expanded to first order to illustrate the parameter dependencies and their degeneracy in the luminosity function. The mass-density constraint is introduced and the redshift dependence of the apparent $\Omega_e(z)$, which is a function of the true Ω , is shown in section 3. In Section 4 the random errors and data requirements of practical measurements are evaluated, concluding that a high precision measurement is primarily a matter of assembling the appropriate datasets, which is likely to be done anyway for a variety of other purposes.

2. Distances and Volumes

The co-moving distance, $r(z)$, to an object at redshift z in an FRW metric is found by integrating radially outward along the null geodesic, $c dt = a(t) dr / \sqrt{1 + r^2/R^2}$, where $a(t) = (1 + z)^{-1}$ is the expansion factor and $R^{-2} = \Omega_R H_0^2 / c^2$ (which is positive in this metric for an open, negatively curved, universe). The integral is rewritten in terms of the observable redshift using the cosmological equation $H(z) = H_0 E(z)$, where $E^2(z) = \Omega_M(1 + z)^3 + \Omega_R(1 + z)^2 + \Omega_\Lambda$ with $\Omega_M + \Omega_\Lambda + \Omega_R \equiv 1$ (following the notation of Peebles 1993). The resulting co-moving distance is,

$$r(z) = \frac{c}{H_0} \frac{1}{|\Omega_R|^{1/2}} \text{sinn} \left[|\Omega_R|^{1/2} \int_0^z \frac{dz'}{E(z')} \right]. \quad (1)$$

The function $\text{sinn}(x)$ is x for $\Omega_R = 0$, $\sin(x)$ for $\Omega_R < 0$, and $\sinh(x)$ for $\Omega_R > 0$ (Carroll, Press & Turner 1992). The co-moving volume element per dz and per steradian is $r^2 dr/dz$, which with the local Hubble law, $H(z) dr (1 + z)^{-1} = c dz (1 + z)^{-1}$, becomes,

$$\frac{dV}{dz} = \frac{c}{H_0} \frac{r^2(z)}{E(z)}. \quad (2)$$

It is useful to note the first order expansions (relative to a Euclidean background) of the co-moving distance,

$$r(z) \simeq \frac{c}{H_0} z \left[1 + \frac{1}{4} (-2 - \Omega_M + 2\Omega_\Lambda) z \right] \quad (3)$$

and the co-moving volume element,

$$\frac{dV}{dz} \simeq \frac{c^3}{H_0^3} z^2 \left[1 + (-2 - \Omega_M + 2\Omega_\Lambda) z \right]. \quad (4)$$

The low redshift deceleration parameter is $q_0 = \Omega_M/2 - \Omega_\Lambda$ and can be used to simplify these first order expansions. There are several noteworthy points to take from these expressions. First, the volume-redshift relation has a sensitivity to the Ω parameters that is twice of the magnitude-redshift, $(1+z)^2 r^2(z)$, relation. Second, the dependence on the cosmological parameters of the distance and the volume element is identical, meaning that luminosity evolution and geometry (or density evolution) are degenerate at this order (which remains approximately true over a wide range of redshifts). Furthermore the test requires a comparison of objects at different redshifts. This generally requires an absolute comparison of fully calibrated quantities, along with all their selection effects.

3. The Apparent Ω

Here we propose a test which is completely differential in the observational quantities: galaxies are only compared to one another at the same redshift and only redshift independent quantities (except for the cosmological parameters of interest) are compared at different redshifts. The co-moving mass density of the universe is an invariant for conventional cosmologies. One estimator of the mass density is through Oort's method, $\Omega_e(z) = M/L \times j/\rho_c$, where M/L is the total mass-to-light ratio of the universe and $j(z)$ is the average field luminosity per unit volume. In the interval $[z, z + \Delta z]$ and solid angle $\Delta\omega$,

$$j(z) = 4\pi(1+z)^2 r^2(z) \frac{\sum_{\Delta z \Delta \omega} f}{\Delta z \Delta \omega dV/dz}, \quad (5)$$

where the f are the observed fluxes of the field galaxies in this volume. The virial mass, or any other dynamical estimator of the gravitational mass (including the mean lensing gravitational shear within an aperture, Kaiser & Squires 1993), is of the form $M = 3G^{-1} \sigma_1^2 \theta_v r(z) (1+z)^{-1}$. The quantity θ_v is the angular scale radius of the cluster, such as either the classical pointwise virial radius estimator or a ringwise estimator (Peebles 1971, Carlberg, Yee & Ellingson 1997). The total cluster luminosity is $L = 4\pi(1+z)^2 r^2(z) \sum_c f$, where the sum adds the fluxes of the galaxies in the redshift and angular range that define the cluster, and is limited at the same f or absolute luminosity as the field galaxies. The apparent density parameter is then,

$$\Omega_e(z) = \left[\frac{\sum_{\Delta z \Delta \omega} f}{\Delta z \sum_c f} \frac{3\sigma_1^2 \theta_v}{G(1+z)\Delta\omega} \right] \frac{H_0}{c\rho_c} \frac{E(z, \Omega^i)}{r(z, \Omega^i)}, \quad (6)$$

where Ω^i are some convenient, but not necessarily correct, values used to calculate the relation. Note that all the observational information is contained between the square brackets. If we assume $\Omega_M = \Omega_M^i$, $\Omega_\Lambda^i = 0$, the resulting effective $\Omega_e(z)$ is then, to first order,

$$\Omega_e(z) \simeq \Omega_M \left[1 + \frac{3}{4} (\Omega_M^i - \Omega_M + 2\Omega_\Lambda) z \right]. \quad (7)$$

The function $\Omega_e(z)$ has a zero point which gives Ω_M and a redshift dependence $\frac{3}{2}\Omega_\Lambda z$, assuming that the initial value, Ω_M^i , is close to the true Ω_M . This redshift dependence is 25% less sensitive to Ω_Λ than the volume-redshift test.

The general behavior of $\Omega_e(z)$ is shown in Figure 1. The plotted lines assume that we calculated the masses and luminosities using $\Omega_M^i = 0.2$ and $\Omega_\Lambda^i = 0$. For this choice of Ω^i the $\Omega_e(z)$ are functions of the true Ω as,

$$\Omega_e(z) = 0.2 \frac{r(0.2, 0, z)}{r(\Omega_M, \Omega_\Lambda)} \frac{E(\Omega_M, \Omega_\Lambda)}{E(0.2, 0, z)} \quad (8)$$

It is clear from this plot that the expected variation of this quantity between redshift zero and unity is sufficient to extract both Ω_M and Ω_Λ . Moreover, the flat, $\Omega_M + \Omega_\Lambda = 1$, models have a distinctly different behavior than open models, for low values of Ω_M .

4. Error Analysis

A practical implementation depends on having sufficient data that the random errors in the result are reduced to the desired level. The data must also allow for checks for systematic errors, notably differential evolution between clustered and field galaxies and whether clustered systems have any segregation between their luminosity and their mass distributions.

4.1. Random errors in $\Omega_e(z)$

In the following analysis we will consider data which uniformly cover the zero to unity redshift range, which is nearly optimal for the application of this test. A smaller redshift range does not give much leverage for the redshift dependence of $\Omega_e(z)$ which is essential for Ω_Λ measurement. On the other hand, pushing the redshift range beyond redshift unity is quite difficult, since many of the spectral features used to measure accurate velocities, in particular the H+K lines and the 4000Å break move out of the region accessible to high efficiency optical spectrographs.

The random errors in estimating the M/L ratio of a single virialized cluster are straightforward to evaluate. For the dynamical estimator of the form given above we need to estimate σ_1^2 , θ_v and L . If the errors are uncorrelated then the fractional error of a single cluster will be approximately $\sqrt{6/N}$. This expectation is borne out quite accurately in available data (Carlberg *et al.* 1996) in which N varies from about 25 to nearly 200. Once N becomes much larger than 100 the statistical error continues to decline in the expected manner but the total error is dominated by projection effects and the internal substructure of the cluster. Furthermore at about a magnitude below M_* in the cluster the field galaxies begin to overwhelm the cluster galaxies. When these limits are encountered, it is better to spread the observations over more clusters rather than continuing to observe the same cluster. This is doubly true since more velocities usually require observing more deeply into the luminosity function where the fraction of the galaxies observed that are in the cluster, as opposed to the field, is an ever declining fraction. In summary, it is readily feasible to obtain M/L values of individual clusters accurate to 25%, which formally requires 96 cluster

members distributed over the face of the cluster. This is only practical for very rich clusters of galaxies.

The number of clusters required for a confident measurement of Ω_Λ is easily evaluated with Monte Carlo simulations of the sample properties. Figure 3 shows the results of 1000 simulations of a sample of 20 clusters that are randomly but uniformly distributed over the $0 \leq z \leq 1$ interval. The cosmology used to generate the distribution has $\Omega = [0.2, 0.8]$ whereas the $\Omega_e(z)$ are calculated assuming $\Omega^i = [0.2, 0]$. The 1σ confidence range is $0.60 \leq \Omega_\Lambda \leq 0.93$, irrespective of Ω_M . The 99% confidence interval is $0.19 \leq \Omega_\Lambda \leq 1.09$, and many of the extreme values result primarily from unusually poor random distributions in redshift, which could be readily avoided in real observations.

To increase the precision of the result requires a larger cluster sample. One hundred clusters, with the same redshift and error distribution as above, can reduce the error in Ω_Λ to a 7% 1σ error. Groups found in a field survey can also be used, however, in that case the errors in M/L for a single group are quite large. An efficient use of the data will be to average the groups together to build up pseudo-clusters of about 500-1000 galaxies which decreases the substructure and projection effects to a level where the profiles of galaxy density and mass can be checked for variation with redshift as well as the changes in galaxy population with redshift. Since between 1/10 and 1/3 of galaxies are in field groups this implies that a field survey of about 20,000 galaxies is in hand. In either case, these surveys are easily feasible with existing instrumentation and will become easier with new facilities.

4.2. Differential Luminosity Evolution

If there was no differential evolution between the galaxies used to estimate M/L and the average over the universe, then luminosity and density evolution would have no impact on our measurement of $\Omega_e(z)$. The virialized systems that will be used range from groups, which contain galaxies quite similar to field galaxies, to rich clusters, which feature far more E and S0 galaxies, having redder colors, than the field galaxy population. Over the $z < 1$ redshift range under discussion all low redshift galaxies (of high surface brightness) have a parent (or possibly several) at higher redshift hence the accounting for mass and luminosity evolution is not confused by completely new galaxies appearing in abundance. For both luminosity evolution and density evolution the most difficult parameter to determine is the characteristic luminosity, M_* . Color differences, which track luminosity evolution (Larson & Tinsley 1978), can be measured to much higher precision.

The pioneering work on faint galaxy evolution established that the brightest galaxies do not evolve much more than a minimal passive evolution of (Koo & Kron 1992, Ellis *et al.* 1996) as expected for bright cluster galaxies (Bower, Lucey & Ellis 1991, Smail *et al.* 1997, Stanford, Eisenhardt & Dickenson 1997). Various observations continue to bear out the basic slow evolution

situation (Schade *et al.* 1996a, 1996b) in spite of the significant changes of both cluster and field populations at fainter absolute magnitudes (Butcher & Oemler 1984, Lilly *et al.* 1995, Ellis *et al.* 1996, Lin *et al.* 1997, Dressler *et al.* 1997, Smail *et al.* 1997). The complications of differential evolution can be greatly minimized by restricting the sample to galaxies more luminous than about $M_* + 1$ mag. Furthermore, by fully sampling the volume of the virialized cluster or group one obtains a sample of cluster galaxies that is closer to the field population, both in range of colors and morphologies, than the central E/S0 galaxy population.

For measurements involving the comparison of galaxies in rich clusters to the field there will be some differential evolution, which we parameterize as $\Delta(j/L, z) = \Delta_0^{jL} + z\Delta_1^{jL}$. As emphasized above, the Δ^{jL} are small compared to the evolution in the j and L themselves, for suitably chosen samples. The straightforward way to determine the Δ^{jL} is through fitting the luminosity functions, $M_*(z) = M_*(0) + z\Delta_M$ to cluster and field galaxies individually, then Δ_1^{jL} is the difference between Δ_M in clusters and the field. It is important to note that the values of the Δ_M will depend on the assumed Ω , however the quantities are only used to find the difference in characteristic magnitudes at the same redshift. The Δ^{jL} have no Ω dependence, being just flux ratios at a common redshift.

The data gathered for the $\Omega_e(z)$ analysis will also be used to measure the luminosity function relative to the field galaxies. Although one should strive to make this an absolute luminosity function with well defined sample criteria, its primary use is in comparing clustered galaxies to field galaxies so sample selection effects that are in common will make no difference to the difference between the two luminosity functions. The maximum likelihood technique of Sandage, Tammann & Yahil (1972). A dataset of 2000 absolute magnitudes is generated from a Schechter luminosity function, $\phi(L, z) = (L/L_*)^{-\alpha} \exp(-L/L_*)$. As a reasonable match to the available data $M_*(z) = M_*(0) + z\Delta M_z$ mag, with $M_*(0) = -20$, $\Delta M_z = -0.7$ and $\alpha = -1$. We assume that the data extend to $M = -19$ mag, although the precise depth makes little difference providing it is 1 to 2 magnitudes below M_* . The 68, 90 and 99% error ellipses are shown for 3 parameter fits for α , $M_*(0)$, and ΔM_z are shown in Figure 3. From this we conclude that the sample will allow a measurement of M_* accurate to 0.1 mag and ΔM_z accurate to 0.2 mag per unit redshift. Normally the field sample will be larger than the cluster sample, so the errors in measuring the same quantities in the field will be no larger.

The error in estimating ΔM_z , will dominate the error in Ω_Λ because it is more difficult to determine precisely and they are terms that are approximately linear in z . However, for a 20 cluster, 100 galaxies per cluster survey (plus the accompanying field galaxies) we expect that the error in Ω_Λ should be about 2/3 of the error in ΔM_z . This is shown in fact to be borne out fairly accurately in the full nonlinear result, shown in Figure 4.

Differential merging has no effect on the measurement of the cluster luminosity, L , or the field luminosity density, j , unless accompanied by star formation. Measurements of the [OII] line in high luminosity cluster and field galaxies (Balogh *et al.* 1997) find that the star formation

in the field adds little mass to these galaxies, that there is no increase in star formation upon cluster entry, and confirms the well known suppression of star formation in clusters. On the other hand, merging will increase the M_* in a way that could be mistaken for luminosity evolution. The colors and morphological types of galaxies, both suitably adjusted for fading in a cluster, can test for luminosity evolution as opposed to merging. The current measurements of the radial change from cluster center to pure field of the mean luminosity and color suggest that relatively little merging of field galaxies relative to cluster galaxies occurs. The dominant effect is that galaxies largely cease forming stars and fade a few tenths of a magnitude when they enter the cluster (Abraham *et al.* 1996, Carlberg, Yee & Ellingson 1997, Balogh *et al.* 1997). The accuracy to which differential luminosity evolution and differential merging can be determined in a multi-color survey is mainly the precision to which the characteristic M_* can be measured, which we have taken as our error estimate. The differential luminosity evolution is already known to be no more than half of the expected variation of $\Omega_e(z)$ (if the universe is flat). Hence, the galaxy sample will be sufficient to increase the precision of the differential evolution measurement to allow confident Ω_Λ estimation.

5. Conclusions

The classical volume-redshift test, which depends upon an absolute comparison of galaxy numbers or luminosities at different redshifts, can be modified to create a much more reliable, completely differential, test. The extra ingredient is to combine quantities which together give the mean co-moving mass density of the universe, which is a conserved quantity. This benefit comes at the modest cost of a 25% reduction in Ω_Λ sensitivity. Monte Carlo simulations show that a sample of 2000 cluster galaxies and a comparable field sample will be able to tightly constrain the differential evolution between cluster and field and will measure Ω_Λ to a precision of about 25%. Moreover, groups of galaxies found within a large field survey will serve the same purpose and provide a second avenue to address differential evolution between cluster and field. Differential evolution between clustered and field galaxies will be addressed using multi-color photometry and imaging. In the longer term, a survey of 100 or so rich clusters will increase the precision of the geometry measurement to about 7%. Such data can also give extremely precise measurements of the evolution of the sample galaxies, although these should not be taken as absolute evolutionary measurements unless care is taken to avoid redshift dependent selection effects.

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Fig. 1.— The variation of the apparent $\Omega_e(z)$ with redshift, where we have assumed that $\Omega_M^i = 0.2$ and $\Omega_\Lambda^i = 0$. The true values of Ω_M range from 0.1 to 0.9 (top to bottom) for $\Omega_\Lambda = 0$ models (dashed lines) and flat, $\Omega_M + \Omega_\Lambda = 1$, models (dotted lines).

Fig. 2.— The 68, 90 and 99% confidence contours in $[\Omega_M, \Omega_\Lambda]$ for a sample of 20 clusters between redshift 0 and 1 in a random uniform distribution, with 25% statistical errors in their $\Omega_e(z)$ values. The true model is $[0.2, 0.8]$. This sample would contain 2000 cluster galaxies. A greater than 99% confidence discrimination between flat and open low density models can be made.

Fig. 3.— The 68, 90 and 99% confidence contours in $[M_*, \Delta M_*]$ for the measurement of the luminosity function and its evolution. The error in determining ΔM_* is about 0.2 mag.

Fig. 4.— The effect of errors in the rate of differential evolution of cluster and field galaxies. The error in Ω_Λ is approximately two-thirds of the error in measuring the Δ_M .

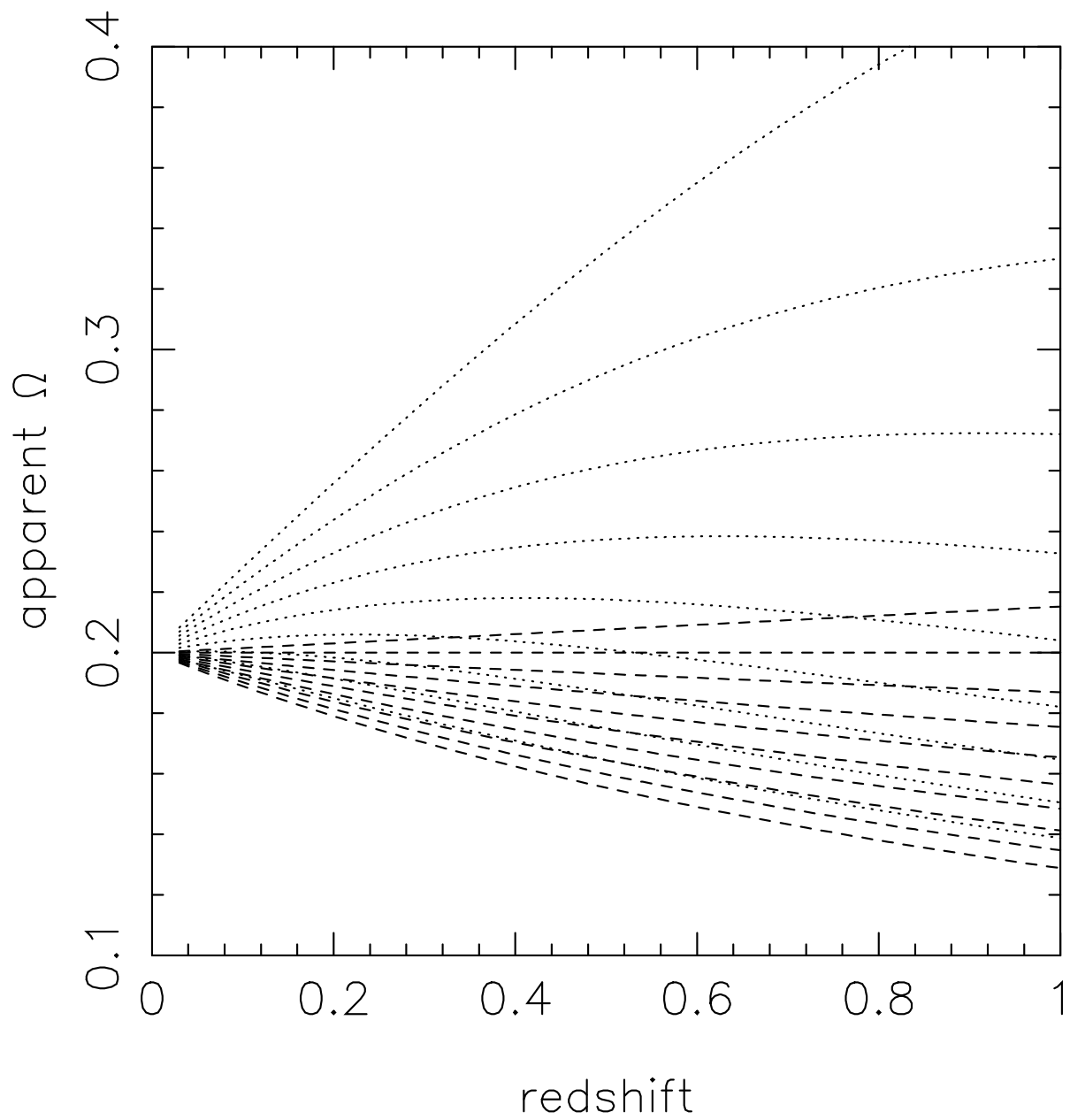


Fig. 1.—

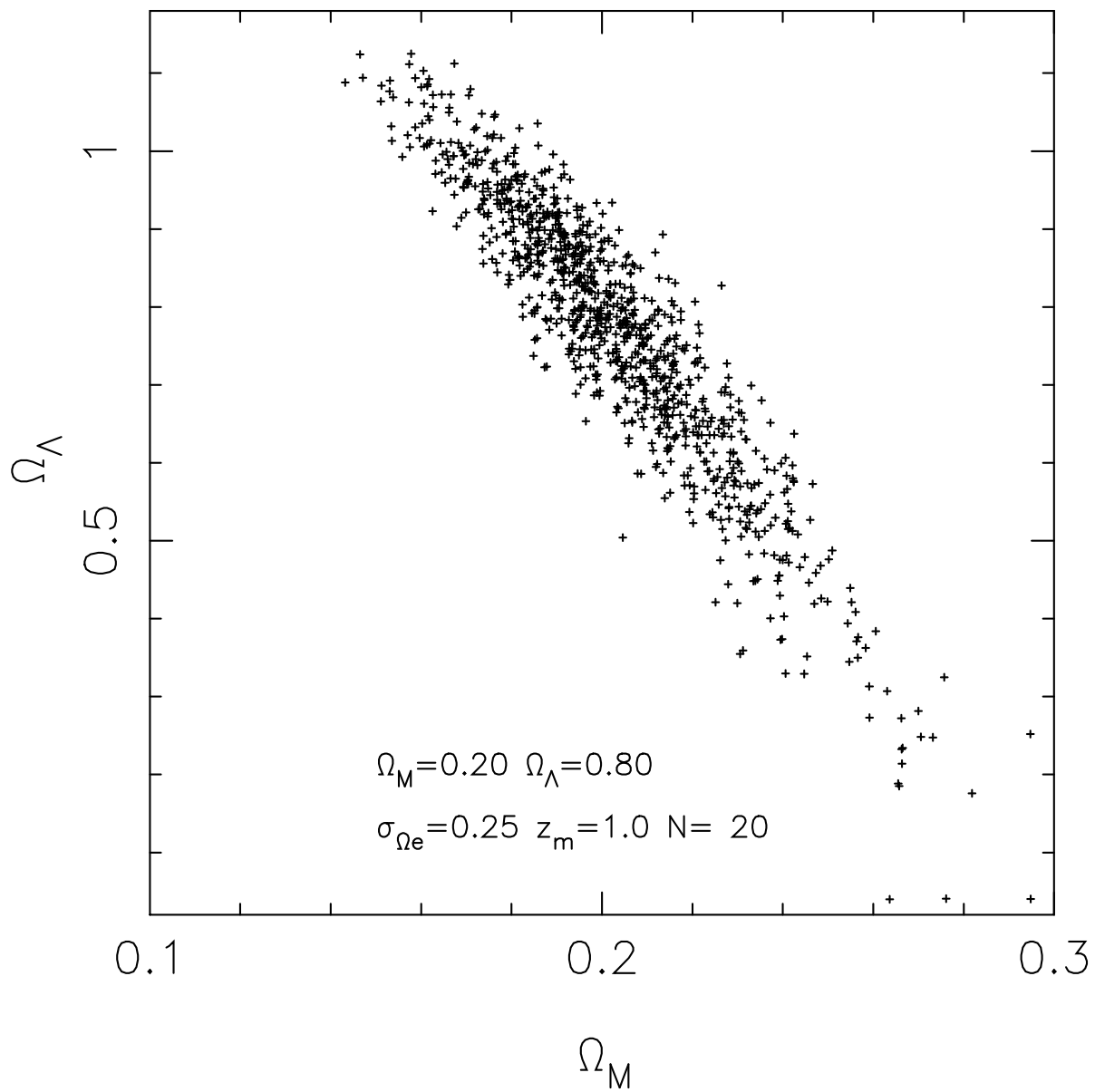


Fig. 2.—

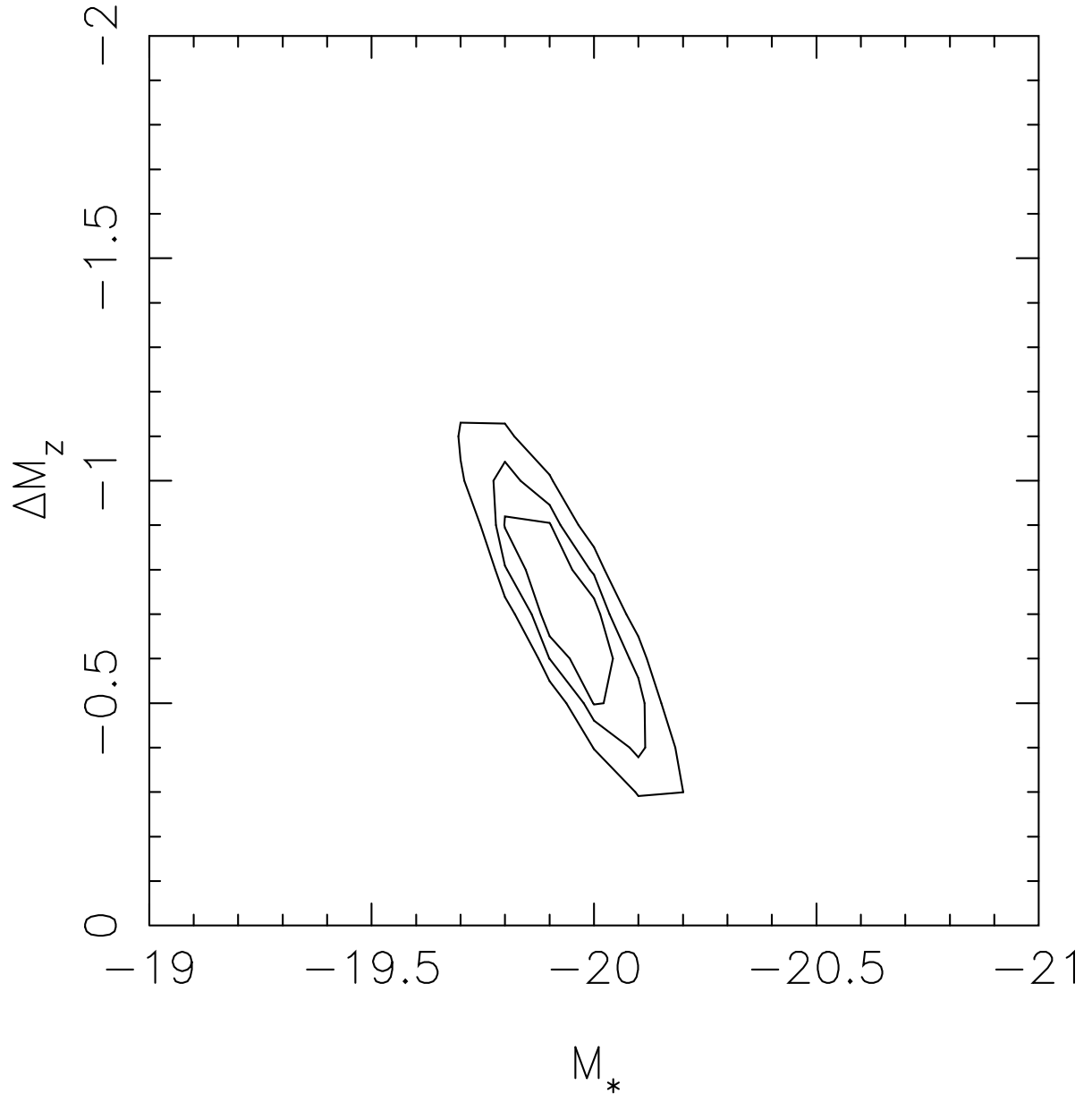


Fig. 3.—

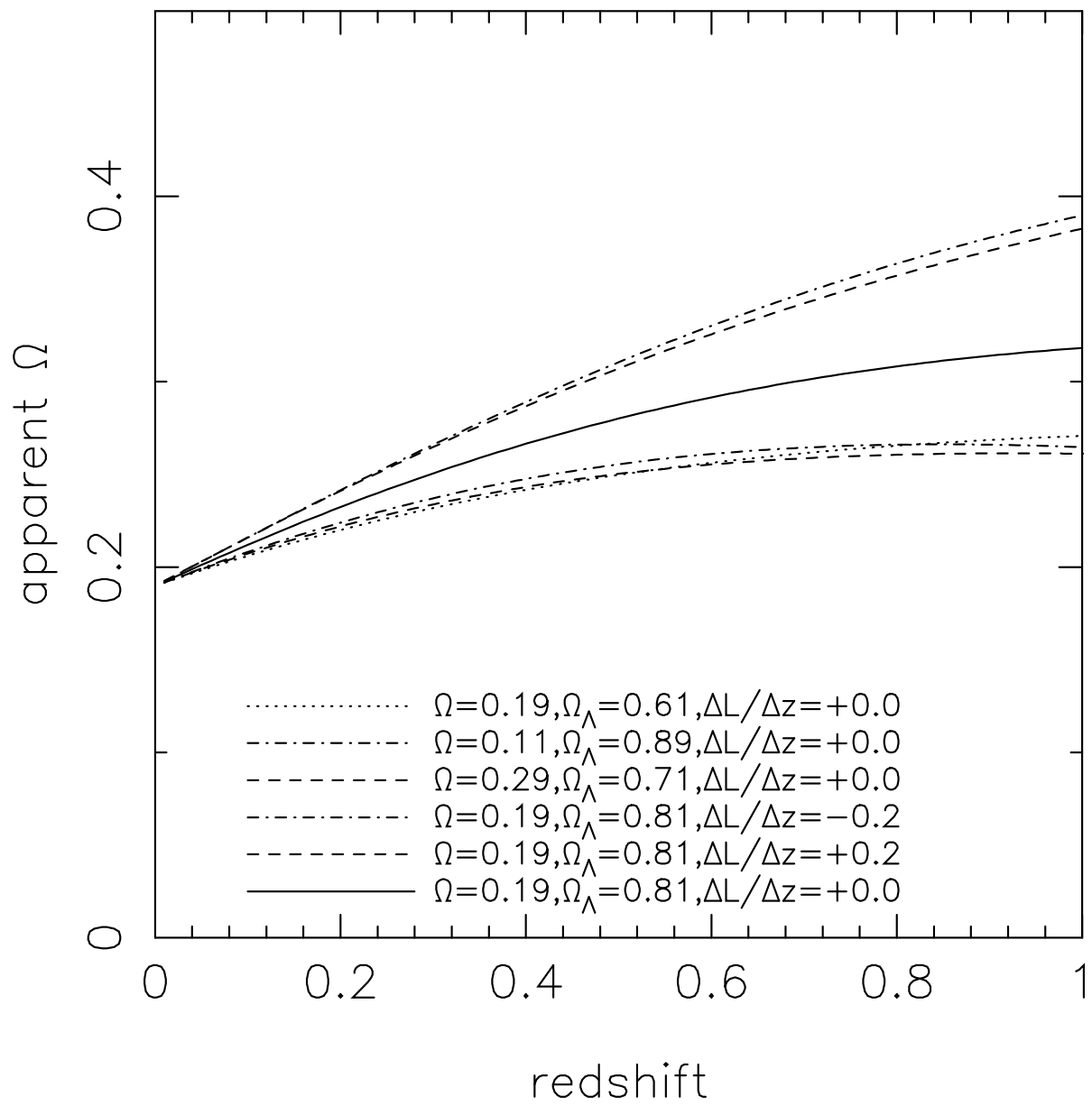


Fig. 4.—